A modal approach to light emission and propagation in coupled cavity waveguide systems

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Abstract— We theoretically investigate systems of optical cavities coupled to waveguides, which necessitates the introduction of non-trivial radiation conditions and normalization procedures. In return, the approach provides simple and accurate modeling of Green functions, Purcell factors and perturbation corrections, as well as an alternative approach to the so-called coupled mode theory. In combination, these results may form part of the foundations for highly efficient, yet physically transparent models of light emission and propagation in both classical and quantum integrated photonic circuits.

Integrated systems of coupled optical waveguides and cavities hold a great potential for future classical and quantum signal processing. Theoretically, the properties of resonant but leaky cavities can be described in terms of quasinormal modes (QNMs), defined as solutions to the electromagnetic wave equation subject to a suitable radiation condition, which depends on the physical system. For cavities embedded in a homogeneous background, the radiation condition is the Silver-Müller condition, which causes the solutions to behave as outgoing waves at large distances. For cavities coupled to waveguides, one can formulate a radiation condition by demanding that the solutions behave as waveguide modes traveling in the directions *away* from the cavity [1, 2]. The left panel in Fig. 1 shows an example of a QNM in a photonic crystal cavity side coupled to a waveguide [2]. The radiation condition results in a discrete spectrum of complex resonance frequencies, $\tilde{\omega}_\mu = \omega_\mu - \mathrm{i}\gamma_\mu$, from which the quality factors can be obtained as $Q_\mu = \omega_\mu/2\gamma_\mu$. The radiative behavior in combination with the complex resonance frequency results in an exponential divergence of the fields in the waveguides, which complicates the formulation of a suitable QNM normalization. Nevertheless, one can regularize the normalization integral via the theory of divergent series [2] and use the QNMs for perturbation calculations [2] as well as simple yet accurate models for the Purcell factor [2, 3]. The right panel in Fig. 1 shows the enhancement in the local density-of-states (LDOS) in the cavity center as a function of frequency [3]. At resonance, the LDOS enhancement equals the Purcell factor as calculated from the Q-value and the effective mode volume [2].

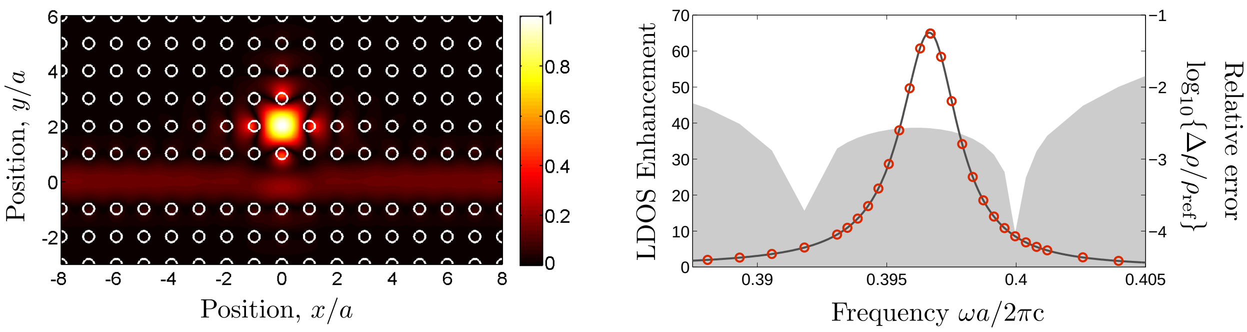

Figure 1: Left: Absolute value of the QNM in a cavity side-coupled to an infinite waveguide in a photonic crystal with lattice constant a. Right: Spectrum of LDOS enhancement for a z-oriented dipole positioned in the center of the cavity. Black solid curve and red circles (left axis) show the QNM approximation ρ_{QNM} and independent reference calculation ρ_{ref}, respectively. Gray shading (right axis) shows the relative error in the single QNM approximation as calculated from the difference $\Delta\rho = \rho_{\mathrm{QNM}} - \rho_{\mathrm{ref}}$.

For applications in classical and quantum optical calculations, the Green function conveniently describes the propagation of light between two different positions. For coupled cavity-waveguide systems, this calls for a hybrid formulation, in which light in the cavities is described in terms of QNMs, and light in the waveguides is treated in terms of waveguide modes. The two families of modes can be coupled using the field equivalence principle, by which an incident field through the waveguide is known to be identical to the field from certain electric and magnetic current sources in a plane intersecting the waveguide. In this way, the complex coupling of light from the waveguide to the cavity can be approximated to high accuracy from properties of the QNMs only [4]. In a simple and illustrative one dimensional example, Fig. 2 shows a comparison between the field in the cavity center as calculated based on QNMs and by traditional scattering methods. In the time domain, this naturally provides an alternative approach to the so-called temporal coupled mode theory, which is typically derived from energy conservation arguments [5], and to which the formulation reduces in the limit of high Q-values [4].

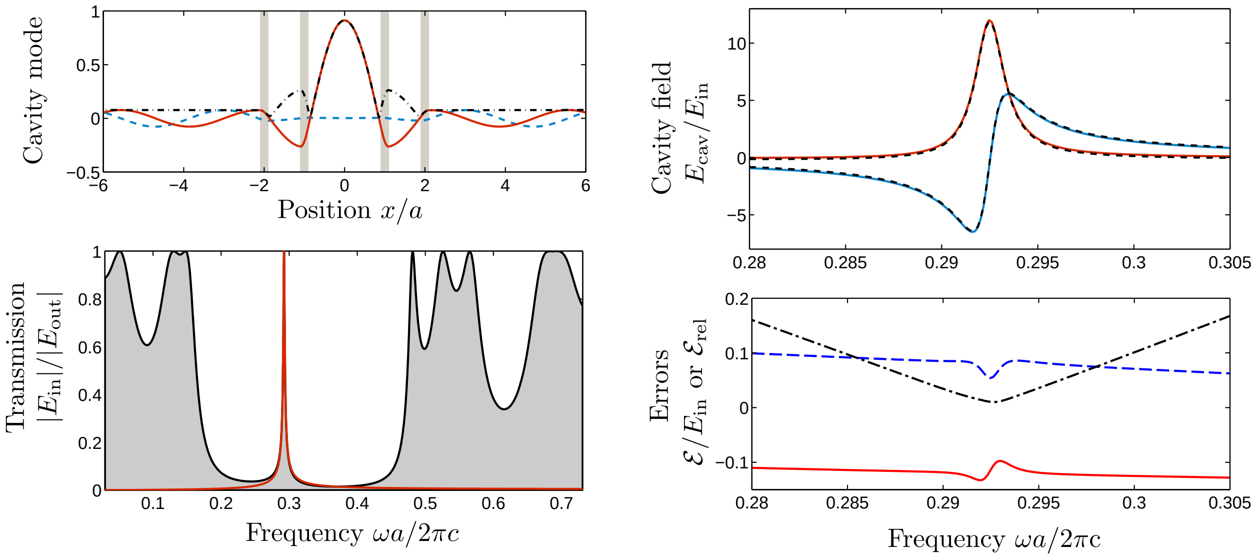

Figure 2: Left top: Real (solid red), imaginary (dashed blue) and absolute (dashed-dotted black) value of the normalized cavity mode of interest in a finite sized one-dimensional photonic crystal (indicated in gray). Left bottom: Transmission spectrum of the cavity. Red line indicates the single mode approximation to the transmission, which is the underlying assumption of the QNM formulation. Right top: Real (solid red) and imaginary (blue) parts of the field in the cavity center when illuminated by a plane wave from the left. Dashed black curves show the result of the QNM formulation. Right bottom: Real (solid red) and imaginary (blue) parts of the error $\mathcal{E} = E_{\rm cav}^{\rm QNM} - E_{\rm cav}^{\rm ref}$ between the two calculations in the top panel. Black dashed-dotted curve shows the relative error $\mathcal{E}_{\rm rel} = |E_{\rm cav}^{\rm QNM} - E_{\rm cav}^{\rm ref}|/|E_{\rm cav}^{\rm ref}|$.

In conclusion, we have investigated light emission and propagation in integrated photonic circuits consisting of optical cavities coupled to waveguides. Light in the cavities may be conveniently described by use of QNMs, providing relatively simple but very accurate modeling of Green functions, Purcell factors and perturbation corrections, and an alternative approach to the so-called temporal coupled mode theory. In combination, these results may form part of the foundation for highly efficient, yet physically transparent models of light emission and propagation in both classical and quantum integrated photonic circuits.

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