Comparison of four computational methods for computing Q factors and resonance wavelengths in photonic crystal membrane cavities

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Abstract – We benchmark four state-of-the-art computational methods by computing quality factors and resonance wavelengths in photonic crystal membrane L5 and L9 line defect cavities. The convergence of the methods with respect to resolution, degrees of freedom and number of modes is investigated. Special attention is paid to the influence of the size of the computational domain. Convergence is not obtained for some of the methods, indicating that some are more suitable than others for analyzing line defect cavities.

The photonic crystal (PhC) membrane represents a platform for planar integration of components, where cavities and waveguides may play a key role in realizing compact optical components with classical functionality[1] such as switches, lasers[2], and amplifiers or quantum optical functionality such as integrated sources of quantum light[3]. By leaving out a row of holes in an otherwise perfect PhC membrane lattice, a line defect is created in which light may be guided. If the waveguide is terminated at both ends, the finite-length waveguide forms an L*n* cavity, where *n* denotes the length of the cavity. Such L*n* cavities support spectrally discrete optical modes, and the fundamental cavity mode profile of an L9 cavity is shown in Fig. 1. Light may be confined to such an L*n* cavity for extended periods, as quantified by the quality (Q) factor. For laser applications, the Q factor governs the onset of lasing, and for cavity quantum electrodynamics applications, it governs the onset of strong coupling. The Q factor thus represents a key parameter in the design of a PhC membrane cavity.

The combination of the large size of the PhC L*n* cavity and the full 3D nature of the geometry makes the

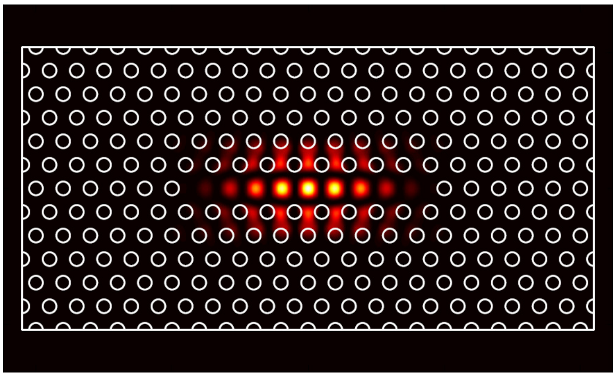

Fig. 1: Optical field $|E_y|^2$ profile in the $z = 0$ plane for the fundamental L9 cavity mode.

calculation of the cavity Q factor an extremely demanding numerical challenge. No matter which numerical method is used, careful convergence checks with respect to the degrees of freedom must be made. Additionally, most numerical simulations methods rely on a closed simulation domain, and here the influence of the boundary conditions requires carefully study. A study of PhC nanobeam cavities using four numerical techniques has previously been reported[4], where cavity frequencies and Q factors were investigated as function of structural parameters. While qualitative agreement between the methods was found, quantitative discrepancies were in some cases as large as an order of magnitude, and estimates for the computational error and the influence of the size of the computational domain were not given.

In this work, we focus on two structures, a low-Q L5 cavity and a high-Q L9 cavity. We employ four different computational methods[5], the finite-difference time-domain (FDTD) technique, the finite-element method (FEM), the surface integral equation (SIE) approach and the Fourier modal method (FMM), to compute the cavity Q factor and the resonance wavelength for both structures. For each method, the relevant computational parameters are systematically varied to quantify the computational errors. In particular, we investigate the influence of the size of the computational domain.

Table 1: Calculated Q factors and resonance wavelengths λ.

	FDTD	FEM	SIE	FMM
λ^{L5} (nm)	1567.7	1571	1572	1567
λ^{L9} (nm)	1583	1578	1579	1572
Q^{L5}	1670	1700	1696	1700
Q^{L9}	90,000	105,000	103,000	50,000

The final results summarized in Table 1 show that both the resonance wavelength and the Q factor agree fairly well for the L5 cavity among the four methods. On the other hand, significant deviations are observed for the Q factor in the L9 cavity. Our study highlights the importance of careful convergence checks and systematic estimation of the computational error, both of which are generally missing in the literature.

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